

42 (1) A C 可逆

$$\text{设 } X^{-1} = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$$

$$\begin{aligned} XX^{-1} &= \begin{bmatrix} O & A \\ C & O \end{bmatrix} \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \\ &= \begin{bmatrix} AX_{21} & AX_{22} \\ CX_{11} & CX_{12} \end{bmatrix} \end{aligned}$$

$$\begin{cases} AX_{21} = E \\ CX_{12} = E \\ AX_{22} = O \\ CX_{11} = O \end{cases} \Rightarrow X^{-1} = \begin{bmatrix} O & C^{-1} \\ A^{-1} & O \end{bmatrix}$$

$$(2) (a) \left[\begin{array}{cc|cc} 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \hline 0 & 0 & 2 & 5 \\ 0 & 0 & 1 & 3 \end{array} \right] = A$$

$$\text{由 (1), } A^{-1} = \begin{bmatrix} A_{11}^{-1} & O \\ O & A_{22}^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 3 & -5 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$(2) Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{13} & Y_{14} \end{bmatrix}$$

$$\begin{aligned} YY^{-1} &= \begin{bmatrix} A & B \\ O & C \end{bmatrix} \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \\ &= \begin{bmatrix} AY_{11} + BY_{21} & AY_{12} + BY_{22} \\ CY_{21} & CY_{22} \end{bmatrix} \end{aligned}$$

$$\begin{cases} AY_{11} + BY_{21} = E \\ CY_{22} = E \\ CY_{21} = O \\ AY_{12} + BY_{22} = O \end{cases}$$

$$\Rightarrow Y^{-1} = \begin{bmatrix} A^{-1} & O \\ O & C^{-1} \end{bmatrix}$$

$$(b) \begin{bmatrix} A & O \\ C & B \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} & O \\ -B^{-1}CA^{-1} & B^{-1} \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \hline -1 & 2 & 2 & 3 \\ 1 & -1 & 1 & 3 \end{array} \right] = X$$

$$X = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ 14 & 30 & 3 & -5 \\ -7 & 11 & -1 & 2 \end{bmatrix}$$

$$43 \text{ B } \begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & B^{-1} \\ A^{-1} & 0 \end{bmatrix} = \frac{\begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}^*}{\begin{vmatrix} 0 & A \\ B & 0 \end{vmatrix}} = \frac{\begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}^*}{-|A||B|}$$

$$\Rightarrow \begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}^* = (-1)^4 |A||B| \begin{bmatrix} 0 & B^{-1} \\ A^{-1} & 0 \end{bmatrix}$$

$$A^{-1} = \frac{A^*}{|A|} = \frac{A^*}{2}$$

$$B^{-1} = \frac{B^*}{|B|} = \frac{B^*}{3}$$

$$\begin{bmatrix} 0 & A \\ B & 0 \end{bmatrix}^* = \begin{bmatrix} 0 & 2B^* \\ 3A^* & 0 \end{bmatrix}$$

$$45 \text{ D } \text{由题} \quad P_1 = E_{21}(1), \quad P_2 = E_{23}$$

$$AP_1 = B$$

$$P_2 B = E \Rightarrow B = P_2^{-1}$$

$$AP_1 = P_2^{-1} \Rightarrow A = P_2^{-1} P_1^{-1}$$

$$\because P_2 P_2 = E$$

$$\therefore A = P_2 P_1^{-1} \rightarrow \text{选 D}$$

$$48. (1). \quad P_1 = E_{31}(2)$$

$$P_2 = E_{31} \quad \text{则} \quad P_2^2 = E$$

$$P_1 A P_2^{100} = E_{31}(2) A \cdot E$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 2a_{11}+a_{31} & 2a_{12}+a_{32} & 2a_{13}+a_{33} \end{bmatrix}$$

$$(2) \quad P_1^{100} A P_2^{999}$$

$$= E_{31}(200) A P_2$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 200a_{11}+a_{31} & 200a_{12}+a_{32} & 200a_{13}+a_{33} \end{bmatrix}$$

$$\text{49. (2)} \quad \left[\begin{array}{ccccc|c} 2 & 1 & 0 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 2 & 1 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1 & 1/2 \end{array} \right] \begin{array}{l} 1/2 \quad -1/4 \quad 1/8 \quad -1/16 \quad 1/32 \\ 1/2 \quad -1/4 \quad 1/8 \quad -1/16 \\ 1/2 \quad -1/4 \quad 1/8 \\ 1/2 \quad -1/4 \\ 1/2 \end{array}$$

$$A^{-1} = \begin{bmatrix} 1/2 & -1/4 & 1/8 & -1/16 & 1/32 \\ & 1/2 & -1/4 & 1/8 & -1/16 \\ & & 1/2 & -1/4 & 1/8 \\ & & & 1/2 & -1/4 \\ & & & & 1/2 \end{bmatrix}$$

$$\text{50 (2)} \quad \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 2 & 2 & 1 \\ 1 & -1 & 0 & 1 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1/3 \\ 0 & 1 & 0 & 1/3 \\ 0 & 0 & 1 & -1/3 \end{array} \right] \begin{array}{l} 1/3 \quad 1/6 \quad 2/3 \\ 1/3 \quad 1/6 \quad -1/3 \\ -1/3 \quad 1/3 \quad 1/3 \end{array}$$

$$\text{79 } X = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 2 \\ 1 & -1 & 0 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/3 & 1/6 & 2/3 \\ 1/3 & 1/6 & -1/3 \\ -1/3 & 1/3 & 1/3 \end{bmatrix}$$

$$= \begin{bmatrix} -1/3 & 1/3 & 4/3 \\ 2/3 & 1/3 & 1/3 \\ 2/3 & 5/6 & 4/3 \end{bmatrix}$$

陈永鑫 3260101045

2026.10.20 W6 Homework

52 (1) 由题 $\exists Q$ (Q 可逆) s.t. $AQ = B$

取行列式 $|A||Q| = |B|$

$$|B| = \begin{vmatrix} 1 & a & 2 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & a & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 2-a \end{vmatrix} = 2-a$$

$$|A| = \begin{vmatrix} 1 & 2 & a \\ 1 & 3 & 0 \\ 2 & 7 & -a \end{vmatrix} = \begin{vmatrix} 1 & 2 & a \\ 0 & 1 & -a \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$\therefore |Q| \neq 0$

$$\therefore 2-a=0 \Rightarrow a=2$$

$$(2) \quad A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 0 \\ 2 & 7 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 2 & 2 \\ 1 & 3 & 0 & 0 & 1 & 1 \\ 2 & 7 & -2 & -1 & 1 & 1 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 2 & 1 & 2 & 2 \\ 0 & 1 & -2 & -1 & -1 & -1 \\ 0 & 3 & -6 & -3 & -3 & -3 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 3 & 4 & 4 \\ 0 & 1 & -2 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

设可逆阵 $M \Rightarrow (MA)P = (MB)$

$$M[A|B] = [MA|MB]$$

$$P = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \Rightarrow \begin{cases} a_{11} + 6a_{31} = 3 \\ a_{12} + 6a_{32} = 4 \\ a_{13} + 6a_{33} = 4 \\ a_{12} - 2a_{33} = -1 \\ a_{22} - 2a_{23} = -1 \\ a_{32} - 2a_{33} = -1 \end{cases}$$

$$\Rightarrow P = \begin{bmatrix} 3-6C_1 & 4-6C_2 & 4-6C_3 \\ -1+2C_1 & -1+2C_2 & -1+2C_3 \\ C_1 & C_2 & C_3 \end{bmatrix}$$